JEE	Class – 12th	Topic – Adjoint and Inverse

Adjoint of a Square Matrix

Let $A = [a_{ij}]_{n\times n}$ is a square matrix of order n and let C_{ij} be the cofactor of a_{ij} in the determinant $|A|$. Then, the adjoint of A is denoted by adj(A) and is defined as the transpose of the cofactor matrix.

Note Adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing the signs of the off diagonal elements.

Properties of Adjoint of a Square Matrix

(i) $(adjA)A = A(adjA) = |A| \cdot I_n$

(ii) $|adjA| = |A|^{n-1}$, if $|A| \neq 0$

(iii) $adj(AB) = (adjB)(adjA)$

(iv) If $|A| = 0$, then $(adjA)A = A(adjA) = 0$

(v) $adj(adjA) = |A|^{n-2}A$, where A is a non-singular matrix

(vi) $adj(A^T) = (adjA)^T$

(vii) Adjoint of a diagonal matrix is a diagonal matrix.

(viii) $|adj(adjA)| = |A|^{(n-1)^2}$

(ix) $adj(A^m) = (adjA)^m, m \in N$

(x) $adj(KA) = K(adjA), K \in R$

(xi) $adj(I_n) = I_n$

(xii) $adj0 = 0$

(xiii) Adjoint of symmetric matrix is a symmetric matrix.

(xiv) A is singular $\Rightarrow |adjA| = 0$

(xv) Adjoint of triangular matrix is a triangular matrix.

Inverse of a Matrix

If two square matrices of same order are A and B, for which

$$AB = BA = I_n$$

Then, B is known as inverse of A, i.e.

$$A^{-1} = B$$

If $|A| \neq 0$ i.e. A is non-singular, then

$$A^{-1} = \frac{adjA}{|A|}$$

Properties of Inverse of a Matrix

If A, B and C are three matrices of same order and $|A| \neq 0$, $|B| \neq 0$ and $|C| \neq 0$, then

(i) (a) $AB = AC \Rightarrow B = C$ (left cancellation law)

(b) $BA = CA \Rightarrow B = C$ (right cancellation law)

(ii) (a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(iii) $(A^T)^{-1} = (A^{-1})^T$

(iv) $(kA)^{-1} = \frac{1}{k}A^{-1}$, if $k \neq 0$

(v) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $|A| \neq 0$, then

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(vi) If A is a non-singular matrix, then

$$|A^{-1}| = |A|^{-1} \Rightarrow |A^{-1}| = \frac{1}{|A|}$$

(vii) If A is a symmetric matrix, then A^{-1} is also a symmetric matrix.

(viii) A square matrix is invertible iff it is non-singular and every invertiable matrix possesses a unique inverse.