JEE	Class –11th	Topic – Roots of Quadratic Equation

1. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is

(JEE Main 2020)

(a) $\frac{5}{27}$ (b) $\frac{25}{9}$

(c) $\frac{5}{9}$ (d) $\frac{25}{81}$

Sol. (d) Given equation $9x^2 - 18|x| + 5 = 0$

$\Rightarrow 9|x|^2 - 18|x| + 5 = 0$

$\Rightarrow 9|x|^2 - 15|x| - 3|x| + 5 = 0$

$\Rightarrow 3|x|(3|x| - 5) - 1(3|x| - 5) = 0$

$\Rightarrow |x| = \frac{1}{3}, \frac{5}{3}$

$\Rightarrow x = \pm\frac{1}{3}, \pm\frac{5}{3}$

So, product of roots $= \left(\frac{1}{3}\right)\left(\frac{-1}{3}\right)\left(\frac{5}{3}\right)\left(\frac{-5}{3}\right) = \frac{25}{81}$

2. Let $p, q \in R$. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then

(JEE Main 2019)

(a) $q^2 - 4p - 16 = 0$ (b) $p^2 - 4q - 12 = 0$

(c) $p^2 - 4q + 12 = 0$ (d) $q^2 + 4p + 14 = 0$

Sol. (b) Given quadratic equation is $x^2 + px + q = 0$, where $p, q \in R$ having one root $2 - \sqrt{3}$, then other root is $2 + \sqrt{3}$

[conjugate of $2 - \sqrt{3}$]

So, sum of roots $= -p = 4 \Rightarrow p = -4$

and product of roots $= q = 4 - 3 \Rightarrow q = 1$

Now, from options $p^2 - 4q - 12 = 16 - 4 - 12 = 0$

3. If α and β are the roots of equation $2x^2 - 5x + 7 = 0$, then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$, is

(a) $2x^2 - 25x + 82 = 0$ (b) $2x^2 + 25x + 82 = 0$

(c) $x^2 - 25x + 82 = 0$ (d) None of these

Sol. (a) Since, α and β be the roots of the equation

$2x^2 - 5x + 7 = 0$, then

$\alpha + \beta = \frac{5}{2}$ and $\alpha\beta = \frac{7}{2}$

Now, sum of roots $= (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = \left(\frac{25}{2}\right)$

and product of roots $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$= 6[(\alpha + \beta)^2 - 2\alpha\beta] + 13\alpha\beta$

$= \left[6 \times \left(\frac{25}{4} - 7\right) + \frac{91}{2}\right] = 41$

The required equation is

$\Rightarrow x^2 - \frac{25}{2}x + 41 = 0$

$2x^2 - 25x + 82 = 0$

4. The roots of the equation

$|2x - 1|^2 - 3|2x - 1| + 2 = 0$ are

(a) $\left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$ (b) $\left\{-\frac{1}{2}, 0, \frac{3}{2}\right\}$

(c) $\left\{-\frac{3}{2}, \frac{1}{2}, 0, 1\right\}$ (d) $\left\{-\frac{1}{2}, 0, 1, \frac{3}{2}\right\}$

Sol. Given equation is

$|2x - 1|^2 - 3|2x - 1| + 2 = 0$

Let

$|2x - 1| = t$

$\therefore t^2 - 3t + 2 = 0$

$\Rightarrow (t - 1)(t - 2) = 0 \Rightarrow t = 1, 2$

$\Rightarrow |2x-1| = 1$ and $|2x-1| = 2$

$\Rightarrow 2x-1 = \pm 1$ and $2x-1 = \pm 2$

$\Rightarrow x = 1,0$ and $x = \frac{3}{2}, -\frac{1}{2}$

5. If the roots of the equation $ax^2 + bx + c = 0$ are of the form $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then $(a+b+c)^2$ is equal to

(a) $b^2 - 4ac$ (b) $b^2 - 2ac$

(c) $2b^2 - ac$ (d) Σa^2

We have, $\frac{k+1}{k} + \frac{k+2}{k+1} = -\frac{b}{a}$

and $\frac{k+1}{k} \cdot \frac{k+2}{k+1} = \frac{c}{a}$

From Eq. (i),

$$1 + \frac{1}{k} + 1 + \frac{1}{k+1} = -\frac{b}{a}$$

$$\Rightarrow \quad 2 + \frac{1}{k} + \frac{1}{k+1} = -\frac{b}{a}$$

From Eq. (ii),

$$\frac{k+2}{k} = \frac{c}{a} \Rightarrow 1 + \frac{2}{k} = \frac{c}{a}$$

$$\Rightarrow \frac{2}{k} = \frac{c}{a} - 1 \Rightarrow k = \frac{2a}{c-a}$$

Now, on substituting the value of k in Eq. (iii), we get

$$2 + \frac{c-a}{2a} + \frac{1}{\frac{2a}{c-a} + 1} = -\frac{b}{a}$$

$$\Rightarrow \quad 2 + \frac{c-a}{2a} + \frac{c-a}{a+c} = -\frac{b}{a}$$

$$\Rightarrow \quad \frac{2(2a)(a+c) + (c-a)(c+a) + 2a(c-a)}{2a(a+c)} = -\frac{b}{a}$$

$$\Rightarrow \quad a^2 + c^2 + 6ac = -2ab - 2bc$$

$$\Rightarrow \quad a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = b^2 - 4ac$$

$$\therefore \quad (a+b+c)^2 = b^2 - 4ac$$

6. If one root of the equation $x^2 + (1 - 3i)x - 2(1 + i) = 0$ is $-1 + i$, then the other root is

(a) $-1 - I$ (b) $\frac{-1-i}{2}$

(c) I (d) 2i

Let another root of equation

$x^2 + (1 - 3i)x - 2(1 + i) = 0$. is α

$\Rightarrow \alpha + (-1 + i) = -(1 - 3i)$

$\Rightarrow \alpha = 2i$

7. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(JEE Main 2019)

(a) 100 (b) 144

(c) -81 (d) -300

Given quadratic equation is

$81x^2 + kx + 256 = 0$

Let one root be α, then other is α^3.

Now, $\alpha + \alpha^3 = -\frac{k}{81}$ and $\alpha \cdot \alpha^3 = \frac{256}{81}$

$\left[\because \text{ for } ax^2 + bx + c = 0, \text{ sum of roots } = -\frac{b}{a}\right.$

$\left.\text{and product of roots } = \frac{c}{a}\right]$

$\Rightarrow \alpha^4 = \left(\frac{4}{3}\right)^4 \Rightarrow \alpha = \pm\frac{4}{3}$

$\therefore k = -81(\alpha + \alpha^3) = -81\alpha(1 + \alpha^2)$

$= -81\left(\pm\frac{4}{3}\right)\left(1 + \frac{16}{9}\right) = \pm 300$