JEE	Class – 11th	Topic – Adjoint and Inverse

Roots of an Equation

The values of a variable in an equation which satisfied the given equation are known as roots of an equation i.e. if $f(x) = 0$ is a polynomial equation and $f(a) = 0$, then a is a root of $f(x) = 0$.

e.g. If $f(x) = x^2 + 3x + 2 = 0$

and $f(-1) = (-1) + 3(-1) + 2 = 1 + 2 - 3 = 0 \Rightarrow f(-1) = 0$

$\therefore -1$ is a root of $f(x) = 0$.

Quadratic Equation

If $f(x)$ is a quadratic polynomial. Then, $f(x) = 0$ is known as quadratic equation. The general form of a quadratic equation is $ax^2 + bx + c = 0, \forall a, b, c \in R$ or C and $a \neq 0$.

Roots of a Quadratic Equation

Roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$, where $b^2 - 4ac$ is known as discriminant and it is denoted by D.

If in a quadratic equation $ax^2 + bx + c = 0$, a, b, c are complex numbers, then we cannot assign positive or negative sign to the discriminant. If one root of an equation is $a + \sqrt{b}$ or $a + ib$, then another root of an equation will be $a - \sqrt{b}$ or $a - ib$, respectively.

Nature of Roots

(i) The roots are real and distinct, iff $D > 0$.

(ii) The roots are real and equal, iff $D = 0$.

(iii) The roots are complex with non-zero imaginary part, iff $D < 0$.

(iv) The roots are rational, iff a, b, c are rational and D is perfect square.

(v) The roots are of the form $p + \sqrt{q}(p, q \in Q)$, iff a, b, c are rational and D is not a perfect square.

(vi) If $a = 1, b, c \in I$ and the roots are rational numbers, then these roots must be integers.

(vii) If a quadratic equation in x has more than two roots, then it is an identity in x that is $a = b = c = 0$.

Relation between Coefficients and Roots of an Equation

(i) Quadratic Equation If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

Sum of roots, $\alpha + \beta = -\frac{b}{a}$

Product of roots, $\alpha\beta = \frac{c}{a}$

(ii) Cubic Equation If α, β and γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$, then

Sum of roots, $\alpha + \beta + \gamma = -\frac{b}{a}$,

Product of two roots, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

Product of three roots, $\alpha\beta\gamma = -\frac{d}{a}$

Formation of an Equation with Given Roots

(i) Quadratic Equation If α and β are the roots of a quadratic equation, then the equation will be $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

(ii) Cubic Equation If α, β and γ are the roots of a cubic equation, then the equation will be

$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$.