Board –ICSE	Class – 10th	Topic – Operations on Matrices

Addition of Matrices :

Compatibility for addition of matrices :

Two matrices can be added together, if they are of the same order. To add two matrices of the same order means to add the corresponding elements of both the matrices.

e.g. If $A = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$,

then $A + B = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2+3 & 1+2 \\ 5+1 & 6+4 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 6 & 10 \end{bmatrix}$

Subtraction of Matrices :

The same rule and method is used for the subtraction of matrices as is used for the addition of matrices.

i.e. If $A = \begin{bmatrix} 5 & 4 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix}$,

then

$$A - B = \begin{bmatrix} 5 & 4 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5-3 & 4-0 \\ 2-4 & 1-2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -2 & -1 \end{bmatrix}$$

Remember :

1. In addition or subtraction of the matrices, the order of the resulting matrix is the same as the order of matrices added or subtracted.
2. If A, B and C are the matrices of the same order, then :

   (i) $A + B = B + A$

   i.e. addition of matrices is commutative.

   (ii) $A + (B + C) = (A + B) + C$

   i.e. addition of matrices is associative.

   (iii) $A + X = B \Rightarrow X = B - A$

Multiplication of Matrices :

Compatibility for multiplication of matrices :

Two matrices A and B can be multiplied together to get the product matrix AB if, and only if, the number of columns in A (the left hand matrix) is equal to the number of rows in **B** (the right hand matrix).

Let matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Since, the number of columns in A = the number of rows in B = 2.

∴ Product matrix AB is possible.

And, $AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Step 1: Multiply every element of 1st row of matrix A with corresponding element of 1st column of B and add them to get the first element of the 1st row of the product matrix AB.

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\ &= [3 \times 1 + 4 \times 3] \\ &= [15] \end{aligned}$$

Step 2: Multiply every element of 1st row of matrix A with corresponding elements of 2nd column of B and add them to get the second element of the lst row of product matrix AB.

$$\begin{aligned} AB = [3 \quad 4] \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix} &= [15 \quad 3 \times 2 + 4 \times 4] \\ &= [15 \quad 22] \end{aligned}$$

Step 3 : In the similar manner, multiply the elements of 2 nd row of A with corresponding elements of the 1st column of B and get the first element of the second row of AB.

$$\begin{aligned} AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} &= \begin{bmatrix} 15 & 22 \\ 5 \times 1 + 0 \times 3 & \end{bmatrix} \\ &= \begin{bmatrix} 15 & 22 \\ 5 & \end{bmatrix} \end{aligned}$$

Step 4 : Finally, multiply the elements of the second row of matrix A with corresponding elements of second column of matrix B to get the second element of the second row of AB.

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 22 \\ 5 & 10 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ 5 & 10 \end{bmatrix}$$

∴ Product of matrices A and B = AB

$$= \begin{bmatrix} \text{1st row of A} \times \text{1 st column of B} & \text{1st row of A} \times \text{2 nd column of B} \\ \text{2nd row of A} \times \text{1 st column of B} & \text{2nd row of A} \times \text{2 nd column of B} \end{bmatrix}$$