Board –F-9	Class – 9th	Topic – Arithmetic Progression

1 Definition of Arithmetic Progression

An **Arithmetic Progression (A.P.)** is a sequence of numbers in which the difference between any two consecutive terms is always a constant. This constant is called the **common difference** and is usually denoted by d.

Example:

$$2, 5, 8, 11, 14, \ldots$$

Here, the common difference $d = 3$.

2 General Form of an A.P.

If the first term of an A.P. is a and the common difference is d, the sequence can be written as:

$$a,\ a + d,\ a + 2d,\ a + 3d,\ \ldots,\ a + (n - 1)d$$

where:

- a = first term
- d = common difference
- n = number of terms
- a_n = nth term $= a + (n - 1)d$

3 Key Terms

Term	Description
First term (a)	The initial term of the sequence.
Common difference (d)	The fixed difference between consecutive terms ($d = a_2 - a_1$).
nth term (a_n)	The term at position n in the sequence.

4 Formulas in Arithmetic Progression

a. nth Term of an A.P.

$$a_n = a + (n-1)d$$

- a_n: nth term
- a: first term
- d: common difference
- n: term number

b. Sum of First n Terms (S_n)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where S_n is the sum of the first n terms.

5 Properties of Arithmetic Progression

- **Adding or Subtracting a Constant:**
  If the same number is added to or subtracted from each term of an A.P., the resulting sequence is also an A.P. with the same common difference.
- **Multiplying or Dividing by a Constant:**
  If each term is multiplied or divided by a non-zero constant, the sequence remains an A.P.
- **Checking for A.P.:**
  A sequence is an A.P. if the difference between any two consecutive terms is constant.
- **Three Numbers in A.P.:**
  Three numbers x, y, z are in A.P. if $2y = x + z$.

6 Types of A.P.

- **Finite A.P.:** Has a fixed number of terms.
  Example: 3, 7, 11, 15 (4 terms)
- **Infinite A.P.:** Continues indefinitely.
  Example: 1, 4, 7, 10, ...

7 Examples

a. Finding the nth Term

Given: $a = 2$, $d = 3$, $n = 5$

$$a_5 = 2 + (5 - 1) \times 3 = 2 + 12 = 14$$

b. Finding the Sum of n Terms

Given: $a = 2$, $d = 3$, $n = 5$

$$S_5 = \frac{5}{2}[2 \times 2 + (5 - 1) \times 3] = \frac{5}{2}[4 + 12] = \frac{5}{2} \times 16 = 40$$

8 Important Points

- The behaviour of an A.P. depends on the sign of d:
  - If $d > 0$: Sequence increases.
  - If $d < 0$: Sequence decreases.
- If the nth term is a linear expression in n, the sequence is an A.P.
- The number of terms equidistant from the start and end of a finite A.P. will have the same sum.

9 Applications of A.P.

- Arranging seats in rows
- Calculating savings with regular deposits
- Patterns in nature and daily life (e.g., roll numbers, calendar dates)