

Board -F-8

Class - 8th

Topic - Trigonometric Ratios: Detailed Theory

What is Trigonometry?

Trigonometry is a branch of mathematics that studies the relationships between the sides and angles of triangles, especially right-angled triangles. The word “trigonometry” comes from the Greek words *trigonon* (triangle) and *metron* (measure).

Right-Angled Triangle and Its Sides

In a right-angled triangle:

- **Hypotenuse:** The longest side, always opposite the right angle.
- **Opposite Side:** The side opposite to the angle of interest (other than the right angle).
- **Adjacent Side:** The side next to the angle of interest (other than the hypotenuse).

The Six Trigonometric Ratios

Trigonometric ratios are defined as the ratios of the lengths of two sides of a right-angled triangle. These are:

Name	Abbreviation	Formula (with respect to angle θ)
Sine	$\sin \theta$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$
Cosine	$\cos \theta$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
Tangent	$\tan \theta$	$\frac{\text{Opposite}}{\text{Adjacent}}$
Cosecant	$\csc \theta$	$\frac{\text{Hypotenuse}}{\text{Opposite}}$
Secant	$\sec \theta$	$\frac{\text{Hypotenuse}}{\text{Adjacent}}$
Cotangent	$\cot \theta$	$\frac{\text{Adjacent}}{\text{Opposite}}$

Remembering the Ratios: SOH-CAH-TOA

- **SOH:** Sine = Opposite / Hypotenuse
- **CAH:** Cosine = Adjacent / Hypotenuse
- **TOA:** Tangent = Opposite / Adjacent

Reciprocal Relationships

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

How to Identify Sides

Given a right-angled triangle and an angle θ (not the right angle):

- The side opposite θ is called the “opposite side.”
- The side next to θ (but not the hypotenuse) is the “adjacent side.”
- The side opposite the right angle is always the “hypotenuse.”

Example

Suppose in triangle ABC , right-angled at B :

- $AB = 6$ units (perpendicular)
- $BC = 8$ units (base)
- $AC = 10$ units (hypotenuse)

For angle C :

$$\sin C = \frac{AB}{AC} = \frac{6}{10} = 0.6$$

$$\cos C = \frac{BC}{AC} = \frac{8}{10} = 0.8$$

$$\tan C = \frac{AB}{BC} = \frac{6}{8} = 0.75$$

Properties of Trigonometric Ratios

- The value of a trigonometric ratio for a given angle is the same for all right-angled triangles with that angle.
- The ratios depend only on the angle, not on the size of the triangle.

Application of Trigonometric Ratios

Trigonometric ratios are used to:

- Find unknown sides or angles in right-angled triangles.
- Solve real-life problems like measuring heights and distances without direct measurement.

Trigonometric Ratios for Standard Angles

The values of trigonometric ratios for certain standard angles ($0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$) are commonly used and can be found in trigonometry tables.

Key Points to Remember

- Always identify the reference angle before labeling sides.
- Use SOH-CAH-TOA to remember the formulas.
- Trigonometric ratios are only defined for acute angles (less than 90°) in a right-angled triangle at this level.
- The hypotenuse is always the side opposite the right angle and is the longest side.

End of Notes