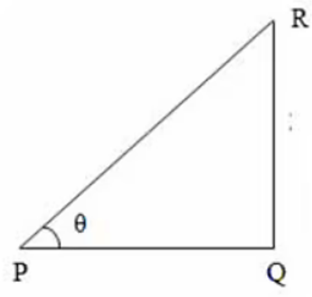
Board –Foundation	Class – 8	Topic – Trigonometric Ratio

1. If $\cos\theta = \frac{3}{5}$, then find the values of $\tan\theta$, $\text{cosec}\theta$.

   Solution: Given, $\cos\theta = \frac{3}{5}$

   Let PQR be the right triangle such that $\angle QPR = \theta$. So,

   $\cos\theta = \frac{PQ}{PR} = \frac{3}{5}$

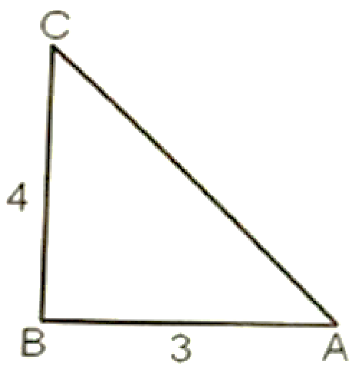
   Assume that $PQ = 3$ and $PR = 5$.

   Then, $QR = \sqrt{PR^2 - PQ^2} = \sqrt{25-9} = 4$

   So, $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{QR}{PQ} = \frac{4}{3}$ and $\text{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{PR}{QR} = \frac{5}{4}$.

2. In $\triangle ABC$, right angled at B, $AB = 3$, $BC = 4$ then determine all trigonometric ratio of angle A.

   Solution: Now by using Pythagoras Theorem

   $AC^2 = AB^2 + BC^2$
   $AC^2 = 3^2 + 4^2$
   $AC = 5$

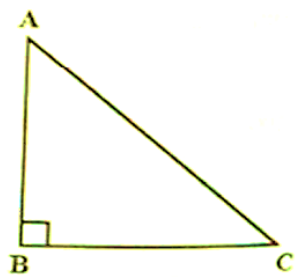
   Now, $\cos A = \frac{AB}{AC} = \frac{3}{5}$

   $\sin A = \frac{BC}{AC} = \frac{4}{5}$ $\quad \cot A = \frac{AB}{BC} = \frac{3}{4}$

   $\tan A = \frac{BC}{AB} = \frac{4}{3}$ $\quad \text{cosec}A = \frac{AC}{BC} = \frac{5}{4}$

   $\sec A = \frac{AC}{AB} = \frac{5}{3}$

3. If θ is an acute angle for $\tan\theta = \frac{5}{12}$, then find the other trigonometric ratios of the angle θ.

   Solution: Let us first draw a right $\triangle ABC$, right angled at $\angle B$. Let one acute angle, $\angle C$ be θ

   $\tan\theta = \frac{AB}{BC} = \frac{5}{12}$

   Therefore, if $AB = 5k$ then $BC = 12k$ where k is positive number

   Now by using the Pythagoras Theorem, we have

   $AC^2 = AB^2 + BC^2 = (5k)^2 + (12k)^2 = 169k^2$ so, $AC = 13k$

   Now we can write all the trigonometric ratio using their definitions.

$\sin\theta = \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$ $\cos\theta = \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\text{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}$

4. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

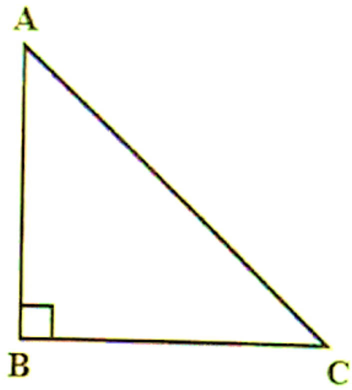
Solution: Let us first draw a right $\triangle$ ABC, right angled at $\angle B$ and let one acute angle, $\angle C$ be θ

Then, $\sec\theta = \frac{13}{12} = \frac{AC}{BC}$

Therefore, if $AC = 13k$ then $BC = 12k$ where k is positive number

$\therefore$ perpendicular $= \sqrt{(13k)^2 - (12k)^2}$

$= \sqrt{(169 - 144)k^2} = 5k$



$\sin\theta = \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{BC}{AC} = \frac{12k}{13k} = \frac{12}{13}$

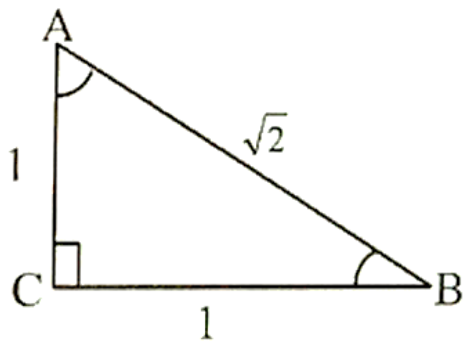
$\tan\theta = \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}$

$\text{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{13}} = \frac{13}{5}$

5. If $\sin A = \frac{1}{\sqrt{2}}$ in right triangle ABC right angled at ' C ', then find the value of $\tan A$, cosecA, $\tan B$, cosecB.

Solution: From the given figure,



$\sin A = \frac{1}{\sqrt{2}} = \frac{BC}{AB} = k$

$\therefore AC = \sqrt{AB^2 - BC^2} = \sqrt{(\sqrt{2}k)^2 - (k)^2}$

$= \sqrt{2k^2 - k^2} = \sqrt{k^2} = k$

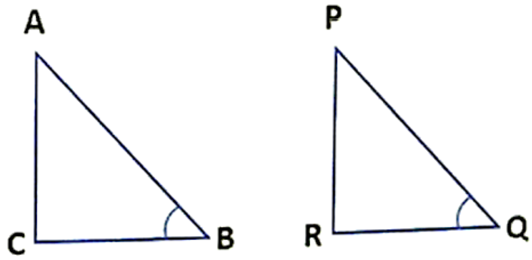
Therefore,

$\tan A = \frac{BC}{AC} = \frac{k}{k} = 1$ $\tan B = \frac{AC}{BC} = \frac{k}{k} = 1$

$\text{cosec}A = \frac{1}{\sin A} = \frac{\sqrt{2}k}{k} = \sqrt{2}$ $\text{cosec}B = \frac{AB}{AC} = \frac{\sqrt{2}k}{k} = \sqrt{2}$

6. If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$. then prove that $\angle B = \angle Q$.

Solution: Let us consider two right triangles ABC and PQR where $\sin B = \sin Q$ (see Fig.).



We have

$\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$

Then $\frac{AC}{AB} = \frac{PR}{PQ}$

$[\because \sin B = \sin Q]$

Therefore,

$\frac{AC}{PR} = \frac{AB}{PQ} = k$, say

Now, using Pythagoras theorem,

$BC = \sqrt{AB^2 - AC^2}$

$QR = \sqrt{PQ^2 - PR^2}$

So, $\frac{BC}{QR} = \frac{\sqrt{AB^2-AC^2}}{\sqrt{PQ^2-PR^2}} = \frac{\sqrt{k^2PQ^2-k^2PR^2}}{\sqrt{PQ^2-PR^2}} = \frac{k\sqrt{PQ^2-PR^2}}{\sqrt{PQ^2-PR^2}} = k$

From (i) and (ii), we have

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

by using theorem which states that " if in two triangles, sides of one triangle are proportional to (i.e.,in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar. $\triangle ACB \sim \triangle PRQ$ and therefore, $\angle B = \angle Q$

7. Consider $\triangle ACB$, right-angled at C, in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$. Determine the values of (a) $\cos^2\theta + \sin^2\theta$, (b) $\cos^2\theta - \sin^2\theta$.

Solution: In $\triangle ACB$, we have $AC = \sqrt{AB^2 - BC^2} = \sqrt{(29)^2 - (21)^2} = \sqrt{(29-21)(29+21)} = \sqrt{(8)(50)} = \sqrt{400} = 20$ units

So, $\sin\theta = \frac{AC}{AB} = \frac{20}{29}, \cos\theta = \frac{BC}{AB} = \frac{21}{29}$.

Now, (a) $\cos^2\theta + \sin^2\theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2 = \frac{21^2+20^2}{29^2} = \frac{441+400}{841} = 1$,

and (b) $\cos^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{(21+20)(21-20)}{29^2} = \frac{41}{841}$.