Board –Foundation	Class –10th	Topic – Operations on Matrices

1. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 4 & -2 \end{bmatrix}$

   find AB and BA if possible.

   Solution:

   Using matrix multiplication. Here, A is a 3×3 matrix, and B is a 3×2 matrix; therefore, A and B are conformable for the product AB , and it is of the order 3×2 such that

   (First row of A) (First column of B)

   $= [2 \quad 13] \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = 2 \times 1 + 1 \times 2 + 3 \times 4 = 16$

   (First row of A) (Second column of B)

   $= [2 \quad 1 \quad 3] \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = 2 \times (-2) + 1 \times 1 + 3 \times (-3) = -12$

   (Second row of A) (First column of B)

   $= [3 \quad -2 \quad 1] \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = 3 \times 1 + (-2) \times 2 + 1 \times 4 = 3$

   $(AB)_{22} = -10, (AB)_{31} = 3$ and $(AB)_{32} = 0$

   $\therefore AB = \begin{bmatrix} 16 & -12 \\ 3 & -10 \\ 3 & 0 \end{bmatrix}$

   BA is not possible since the number of columns of B $\neq$ the number of rows of A .

2. Find the value of x and y if

   $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

   Solution:

Using the method of multiplication and addition of matrices, then equating the corresponding elements of LH.S, and R.H.S. we can easily get the required values of x and y.

We have,

$$2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2 & 6\\ 0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}2+y & 6+0\\ 0+1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

Equating the corresponding elements, a_n and a_{22}, we get

$2+y=5 \Rightarrow y=3; 2x+2=8 \Rightarrow 2x=6 \Rightarrow x=3$

Hence $x=3$ and $y=3$.

3. Find the value of a, b, c and d , if

$$\begin{bmatrix}a-b & 2a+c\\ 2a-b & 3c+d\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 0 & 13\end{bmatrix}$$

Solution:

As the two matrices are equal, their corresponding elements are equal. Therefore, by equating the corresponding elements of given matrices, we will obtain the value of a, b, c and d.

$$\begin{bmatrix}a-b & 2a+c\\ 2a-b & 3c+d\end{bmatrix}=\begin{bmatrix}-1 & 5\\ 0 & 13\end{bmatrix}$$

(given)

$a-b=-1-(i)$

$2a+c=5 \ldots (ii)$

$2a-b=0.. (iii)$

$3c+d=13 \ldots (iv)$

Subtracting equation (i) from (iii), we have $a=1$;

Putting the value of a in equation (i), we get $b=2$

Putting the value of a in equation (ii), we have

$2+c=5 \Rightarrow c=3$

Putting the value of c in equation (iv), we find

$9+x=13 \Rightarrow d=4$

Hence $a=1, b=2, c=3, d=4$.

4. Find x and y, if

$2x+3y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$ and $3x+2y=\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix}$

Solution:

Solving the given equations simultaneously, we will obtain the values of x and y .

We have

$$2x+3y=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix} \ldots \ldots$$

$$3x+2y=\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix}.$$

Multiplying (i) by 3 and (ii) by 2 , we get

$$6x+9y=\begin{bmatrix}6 & 9\\12 & 0\end{bmatrix} \ldots \text{(iii)}$$

$$6x+4y=\begin{bmatrix}4 & -4\\-2 & 10\end{bmatrix} \ldots \text{(iv)}$$

Subtracting (iv) from (iii), we get

$$5y=\begin{bmatrix}6-4 & 9+4\\12+2 & 0-10\end{bmatrix}=\begin{bmatrix}2 & 13\\14 & -10\end{bmatrix}$$

$$\Rightarrow y=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\\frac{14}{5} & \frac{-10}{5}\end{bmatrix} \Rightarrow y=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\\frac{14}{5} & -2\end{bmatrix}$$

Putting the value of y in (iii), we get

$$2x+3\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\\frac{14}{5} & -2\end{bmatrix}=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}$$

$$\Rightarrow 2x=\begin{bmatrix}2 & 3\\4 & 0\end{bmatrix}-\begin{bmatrix}\frac{6}{5} & \frac{39}{5}\\\frac{42}{5} & -6\end{bmatrix}=\begin{bmatrix}2-\frac{6}{5} & 3-\frac{39}{5}\\4-\frac{42}{5} & 0+6\end{bmatrix}=\begin{bmatrix}\frac{4}{5} & -\frac{24}{5}\\-\frac{22}{5} & 6\end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} \text{ and } y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

5. For matrices A and B , subtract matrix B from matrix A

$$A = \begin{bmatrix} 2 & 9 \\ 5 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 7 \\ 2 & 3 \end{bmatrix}$$

Solution:

Matrix A and Matrix B can be easily subtracted as their order is the same. The subtraction of matrix A and matrix B is found as,

$$A - B = \begin{bmatrix} 2-1 & 9-7 \\ 5-2 & 6-3 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}$$

Example: Multiply the matrix **A** by the scalar value $\mathbf{k} = \mathbf{3}$.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The scalar multiplication kA is computed by multiplying each element of A by 3 :

$$kA = 3 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 \times 1 & 3 \times 2 \\ 3 \times 3 & 3 \times 4 \end{bmatrix}$$

So,

$$kA = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$$

6. Let $A = \begin{bmatrix} 1 & 8 & 3 \\ 9 & 4 & 5 \\ 6 & 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 & 4 \\ 1 & 3 & 2 \\ 5 & 9 & 8 \end{bmatrix}$ Find $A \times B$?

Solution:

$$A \times B = \begin{bmatrix} 1 & 8 & 3 \\ 9 & 4 & 5 \\ 6 & 2 & 7 \end{bmatrix} \times \begin{bmatrix} 6 & 7 & 4 \\ 1 & 3 & 2 \\ 5 & 9 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times 6 + 8 \times 1 + 3 \times 5) & (1 \times 7 + 8 \times 3 + 3 \times 9) & (1 \times 4 + 8 \times 2 + 3 \times 8) \\ (9 \times 6 + 4 \times 1 + 5 \times 5) & (9 \times 7 + 4 \times 3 + 5 \times 9) & (9 \times 4 + 4 \times 2 + 5 \times 8) \\ (6 \times 6 + 2 \times 1 + 7 \times 5) & (6 \times 7 + 2 \times 3 + 7 \times 9) & (6 \times 4 + 2 \times 2 + 7 \times 8) \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 58 & 44 \\ 83 & 120 & 84 \\ 73 & 111 & 84 \end{bmatrix}$$