

Board –F-10

Class – 10th

Topic – Operations on Matrices

What is a Matrix?

A **matrix** is a rectangular array of numbers or expressions arranged in rows and columns. Each number in a matrix is called an **element**.

Example of a 2×3 matrix (2 rows, 3 columns):

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Types of Matrix Operations

1. Addition of Matrices

- **Condition:** Matrices must have the same order (same number of rows and columns).
- **How:** Add corresponding elements.

Example:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 7 & 2 \end{bmatrix}$$
$$A + B = \begin{bmatrix} 2+1 & 3+0 \\ 4+7 & 5+2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 11 & 7 \end{bmatrix}$$

2. Subtraction of Matrices

- **Condition:** Matrices must have the same order.
- **How:** Subtract corresponding elements.

Example:

$$A - B = \begin{bmatrix} 2-1 & 3-0 \\ 4-7 & 5-2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -3 & 3 \end{bmatrix}$$

3. Scalar Multiplication

- **How:** Multiply every element of the matrix by a scalar (a constant number).

Example:

$$k = 3, \quad A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$
$$kA = \begin{bmatrix} 3 \times 2 & 3 \times 3 \\ 3 \times 4 & 3 \times 5 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 12 & 15 \end{bmatrix}$$

4. Multiplication of Matrices

- **Condition:** Number of columns in the first matrix = number of rows in the second matrix.
- **How:** Multiply rows of the first matrix by columns of the second matrix and sum the products for each position.

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1 \times 5 + 2 \times 7) & (1 \times 6 + 2 \times 8) \\ (3 \times 5 + 4 \times 7) & (3 \times 6 + 4 \times 8) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

5. Transpose of a Matrix

How: Flip the matrix over its diagonal (rows become columns and vice versa).

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Important Points

- **Order matters:** For addition and subtraction, matrices must be of the same order. For multiplication, the number of columns of the first matrix must equal the number of rows of the second.
- **Matrix multiplication is not commutative:** $AB \neq BA$ in general.
- **Identity Matrix:** A square matrix with 1s on the diagonal and 0s elsewhere.
- **Zero Matrix:** All elements are zero.

Summary Table

Operation	Condition for Operation	How to Perform
Addition	Same order	Add corresponding elements
Subtraction	Same order	Subtract corresponding elements
Scalar Multiplication	Any matrix	Multiply each element by scalar
Matrix Multiplication	Columns of 1st = Rows of 2nd	Row \times Column multiplication
Transpose	Any matrix	Flip rows and columns