Board –CBSE	Class – 9th	Topic – Factorisation of Polynomials

Q1 Examine whether $x+2$ is a factor of x^3+3x^2+5x+6 and of $2x+4$.

Soln. The zero of $x+2$ is -2 . Let $p(x)=x^3+3x^2+5x+6$ and $s(x)=2x+4$

Then,

$$\begin{aligned} p(-2) &= (-2)^3+3(-2)^2+5(-2)+6 \\ &= -8+12-10+6 \\ &= 0 \end{aligned}$$

Again,

$s(-2)=2(-2)+4=0$

So, $x+2$ is a factor of $2x+4$. In fact, you can check this without applying the Factor Theorem, since $2x+4=2(x+2)$.

Q.2 Find the value of k, if $x-1$ is a factor of $4x^3+3x^2-4x+k$.

Soln. As $x-1$ is a factor of $p(x)=4x^3+3x^2-4x+k, p(1)=0$

Now,

$p(1)=4(1)^3+3(1)^2-4(1)+k$

So,

$$\begin{aligned} 4+3-4+k &= 0 \\ k &= -3 \end{aligned}$$

Q.3 Factorise $6x^2+17x+5$ by splitting the middle term, and by using the Factor Theorem.

Soln. (By splitting method) : If we can find two numbers p and q such that $p+q=17$ and $pq=6\times 5=30$, then we can get the factors. So, let us look for the pairs of factors of 30 . Some are 1 and 30,2 and 15,3 and 10,5 and 6 . Of these pairs, 2 and 15 will give us $p+q=17$.

So, $6x^2+17x+5=6x^2+(2+15)x+5$

$$= 6x^2 + 2x + 15x + 5$$
$$= 2x(3x + 1) + 5(3x + 1)$$
$$= (3x + 1)(2x + 5)$$

Sol2 (Using the Factor Theorem)

$6x^2 + 17x + 5 = 6\left(x^2 + \frac{17}{6}x + \frac{5}{6}\right) = 6p(x)$, say. If a and b are the zeroes of $p(x)$, then $6x^2 + 17x + 5 = 6(x - a)(x - b)$. So, $ab = \frac{5}{6}$. Let us look at some possibilities for a and

b. They could be $\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{5}{2}, \pm 1$. Now, $p\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{17}{6}\left(\frac{1}{2}\right) + \frac{5}{6} \neq 0$. But $p\left(\frac{-1}{3}\right) = 0$. So, $\left(x + \frac{1}{3}\right)$ is a factor of $p(x)$. Similarly, by trial, you can find that $\left(x + \frac{5}{2}\right)$ is a factor of $p(x)$.

Therefore,

$$6x^2 + 17x + 5 = 6\left(x + \frac{1}{3}\right)\left(x + \frac{5}{2}\right)$$
$$= 6\left(\frac{3x + 1}{3}\right)\left(\frac{2x + 5}{2}\right)$$
$$= (3x + 1)(2x + 5)$$

For the example above, the use of the splitting method appears more efficient. However, let us consider another example.

Q.4 Factorise $y^2 - 5y + 6$ by using the Factor Theorem.

Soln. Let $p(y) = y^2 - 5y + 6$. Now, if $p(y) = (y - a)(y - b)$, you know that the constant term will be ab. So, $ab = 6$. So, to look for the factors of $p(y)$, we look at the factors of 6 .

The factors of 6 are 1,2 and 3 .

Now, $p(2) = 2^2 - (5 \times 2) + 6 = 0$

So, $y - 2$ is a factor of $p(y)$.

Also, $p(3) = 3^2 - (5 \times 3) + 6 = 0$

So, $y - 3$ is also a factor of $y^2 - 5y + 6$.

Therefore, $y^2 - 5y + 6 = (y - 2)(y - 3)$

Note that $y^2 - 5y + 6$ can also be factorised by splitting the middle term $-5y$.

Now, let us consider factorising cubic polynomials. Here, the splitting method will not be appropriate to start with. We need to find at least one factor first, as you will see in the following example.

Q.5 Factorise $x^3 - 23x^2 + 142x - 120$.

Soln. Let $p(x) = x^3 - 23x^2 + 142x - 120$

We shall now look for all the factors of -120 . Some of these are $\pm 1, \pm 2, \pm 3,$

$\pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24, \pm 30, \pm 60.$

By trial, we find that $p(1) = 0$. So $x - 1$ is a factor of $p(x)$.

Now we see that $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$= x^2(x-1) - 22x(x-1) + 120(x-1)$ (Why ?)

$= (x-1)(x^2 - 22x + 120)$ [Taking $(x-1)$ common]

We could have also got this by dividing $p(x)$ by $x - 1$.

Now $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem. By splitting the middle term, we have:

$$x^2 - 22x + 120 = x^2 - 12x - 10x + 120$$
$$= x(x-12) - 10(x-12)$$
$$= (x-12)(x-10)$$

Q.6 Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Soln. Let $p(x) = x^3 - ax^2 + 6x - a$

$x - a = 0$

$\therefore$ x = a

Remainder:

$p(a) = (a)^3 - a(a^2) + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

Q7. Factorise $y^2 - 5y + 6$ by using the Factor Theorem.

Soln. Let $p(y) = y^2 - 5y + 6$. Now, if $p(y) = (y - a)(y - b)$, you know that the constant term will be ab. So, $ab = 6$. So, to look for the factors of $p(y)$, we look at the factors of 6 .

The factors of 6 are 1,2 and 3 .

Now, $p(2) = 2^2 - (5 \times 2) + 6 = 0$

So, $y - 2$ is a factor of $p(y)$.

Also, $p(3) = 3^2 - (5 \times 3) + 6 = 0$

So, $y - 3$ is also a factor of $y^2 - 5y + 6$.

Therefore, $y^2 - 5y + 6 = (y - 2)(y - 3)$

Note that $y^2 - 5y + 6$ can also be factorised by splitting the middle term $-5y$.

Now, let us consider factorising cubic polynomials. Here, the splitting method will not be appropriate to start with. We need to find at least one factor first, as you will see in the following example.

Q8. Factorise $x^3 - 23x^2 + 142x - 120$.

Soln Let $p(x) = x^3 - 23x^2 + 142x - 120$

We shall now look for all the factors of -120 . Some of these are $\pm 1, \pm 2, \pm 3$,

$\pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24, \pm 30, \pm 60$.

By trial, we find that $p(1) = 0$. So $x - 1$ is a factor of $p(x)$.

Now we see that $x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$ (Why ?)

$= (x - 1)(x^2 - 22x + 120)$ [Taking $(x - 1)$ common]

We could have also got this by dividing $p(x)$ by $x - 1$.

Now $x^2 - 22x + 120$ can be factorised either by splitting the middle term or by using the Factor theorem. By splitting the middle term, we have:

$$\begin{aligned} x^2 - 22x + 120 &= x^2 - 12x - 10x + 120 \\ &= x(x - 12) - 10(x - 12) \\ &= (x - 12)(x - 10) \end{aligned}$$

Q.9 Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Soln. Let $p(x) = x^3 - ax^2 + 6x - a$

$x - a = 0$

$\therefore \mathrm{x} = \mathrm{a}$

Remainder:

$$\begin{aligned} p(a) &= (a)^3 - a(a^2) + 6(a) - a \\ &= a^3 - a^3 + 6a - a = 5a \end{aligned}$$