| Board –CBSE | Class – 10th | Topic – Distance and Section Formula |
|---|---|---|

Q1 Do the points $(3,2), (-2,-3)$ and $(2,3)$ form a triangle? If so, name the type of triangle formed.

Soln. Let us apply the distance formula to find the distances PQ, QR and PR, where $P(3,2), Q(-2,-3)$ and $R(2,3)$ are the given points. We have

$$PQ = \sqrt{(3+2)^2 + (2+3)^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 7.07 \text{ (approx.)}$$

$$QR = \sqrt{(-2-2)^2 + (-3-3)^2} = \sqrt{(-4)^2 + (-6)^2} = \sqrt{52}$$

$$= 7.21 \text{ (approx.)}$$

$$PR = \sqrt{(3-2)^2 + (2-3)^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2} = 1.41 \text{ (approx.)}$$

Since the sum of any two of these distances is greater than the third distance, therefore, the points P, Q and R form a triangle.

Q2 Show that the points $(1,7), (4,2), (-1,-1)$ and $(-4,4)$ are the vertices of a square.

Soln. Let $A(1,7), B(4,2), C(-1,-1)$ and $D(-4,4)$ be the given points. One way of showing that ABCD is a square is to use the property that all its sides should be equal and both its digonals should also be equal. Now,

$$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

Since, $AB = BC = CD = DA$ and $AC = BD$, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Thereore, ABCD is a square.

Q.3 Find a relation between x and y such that the point $(x, y)$ is equidistant from the points $(7,1)$ and $(3,5)$.

Soln Let $P(x, y)$ be equidistant from the points $A(7,1)$ and $B(3,5)$.

We are given that $AP = BP$. So, $AP^2 = BP^2$

i.e.,

$$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$$

i.e.,

$$x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$$

i.e.,

$$x - y = 2$$

which is the required relation.

Remark : Note that the graph of the equation $x - y = 2$ is a line. From your earlier studies, you know that a point which is equidistant from A and B lies on the perpendicular bisector of AB . Therefore, the graph of $x - y = 2$ is the perpendicular bisector of AB (see Fig. 7.7).

Q.4 Find a point on the y-axis which is equidistant from the points $A(6,5)$ and $B(-4,3)$.

Soln. We know that a point on the y-axis is of the form $(0, y)$. So, let the point $P(0, y)$ be equidistant from A and B . Then

$$(6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$$

$$= \left( \begin{array}{rl} 36 + 25 + y^2 - 10y & = 16 + 9 + y^2 - 6y \\ \text{i.e.,} & 4y \\ \text{i.e.,} & 36 \\ \text{i.e.,} & y \end{array} \right.$$

So, the required point is $(0,9)$.

Let us check our solution : $AP = \sqrt{(6-0)^2 + (5-9)^2} = \sqrt{36+16} = \sqrt{52}$

$$BP = \sqrt{(-4-0)^2 + (3-9)^2} = \sqrt{16+36} = \sqrt{52}$$

Note : Using the remark above, we see that (0,9) is the intersection of the y-axis and the perpendicular bisector of AB .

Q.5 In what ratio does the point ( $-4,6$ ) divide the line segment joining the points $A(-6,10)$ and $B(3,-8)$ ?

Soln. Let $(-4,6)$ divide AB internally in the ratio $m_1: m_2$. Using the section formula, we get

$$(-4,6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$$

Recall that if $(x, y) = (a, b)$ then $x = a$ and $y = b$.

So,

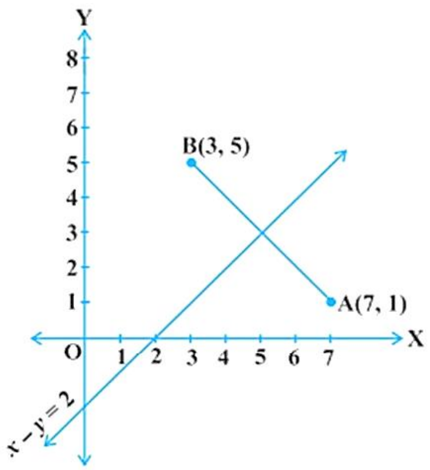
$$-4 = \frac{3m_1 - 6m_2}{m_1 + m_2} \text{ and } 6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$$

Now,

$$-4 = \frac{3m_1 - 6m_2}{m_1 + m_2}$$
$$-4m_1 - 4m_2 = 3m_1 - 6m_2$$
$$7m_1 = 2m_2$$
$$m_1: m_2 = 2: 7$$

Q.6 Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points $A(2,-2)$ and $B(-7,4)$.

Soln. Let P and Q be the points of trisection of AB i.e., $AP = PQ = QB$ (see Fig. ).




Therefore, P divides AB internally in the ratio 1: 2. Therefore, the coordinates of P , by applying the section formula, are $\left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right)$, i.e., $(-1,0)$

Now, Q also divides AB internally in the ratio 2: 1. So, the coordinates of Q are

$$\left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right), \text{ i.e., } (-4,2)$$

Q.7 If the points $A(6,1), B(8,2), C(9,4)$ and $D(p,3)$ are the vertices of a parallelogram, taken in order, find the value of p.

Soln. We know that diagonals of a parallelogram bisect each other. So, the coordinates of the mid-point of AC = coordinates of the mid-point of BD

i.e.,

$$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$$

i.e.,

$$\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

so,

$$\frac{15}{2} = \frac{8+p}{2}$$

i.e.,

$p = 7$