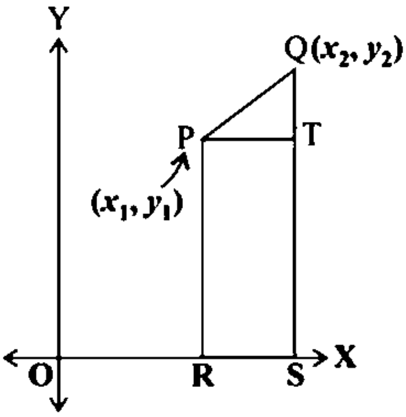
Board –CBSE	Class – 10th	Topic – Distance and Section Formula

Distance Formula:

Let us now find the distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$. Draw PR and QS perpendicular to the x-axis. A perpendicular from the point P on QS is drawn to meet it at the point T (see Fig.).



Then, $OR = x_1, OS = x_2$. So, $RS = x_2 - x_1 = PT$.

Also, $SQ = y_2, ST = PR = y_1$. So, $QT = y_2 - y_1$.

Now, applying the Pythagoras theorem in $\triangle$ PTQ, we get

$$PQ^2 = PT^2 + QT^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Therefore,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note that since distance is always non-negative, we take only the positive square root. So, the distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

which is called the distance formula.

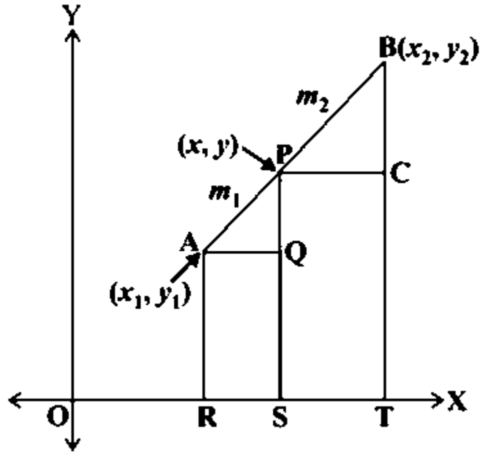
Remarks :

In particular, the distance of a point $P(x, y)$ from the origin $O(0,0)$ is given by

$$OP = \sqrt{x^2 + y^2}$$

Section Formula:

Consider any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ and assume that $P(x, y)$ divides AB internally in the ratio $m_1 : m_2$, i.e., $\frac{PA}{PB} = \frac{m_1}{m_2}$ (see Fig.).



Draw AR, PS and BT perpendicular to the x-axis. Draw AQ and PC parallel to the x-axis. Then, by the AA similarity criterion,

$$\Delta PAQ \sim \Delta BPC$$

Therefore,

$$\frac{PA}{BP} = \frac{AQ}{PC} = \frac{PQ}{BC}$$

Now,

$$AQ = RS = OS - OR = x - x_1$$
$$PC = ST = OT - OS = x_2 - x$$
$$PQ = PS - QS = PS - AR = y - y_1$$
$$BC = BT - CT = BT - PS = y_2 - y$$

Substituting these values in (1), we get

$$\frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

Taking

$$\frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x}, \text{ we get } x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

Similarly, taking

$$\frac{m_1}{m_2} = \frac{y - y_1}{y_2 - y}, \text{ we get } y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

So, the coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

This is known as the section formula.

Special Case : The mid-point of a line segment divides the line segment in the ratio $1:1$. Therefore, the coordinates of the mid-point P of the join of the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\left(\frac{1\cdot x_1+1\cdot x_2}{1+1},\frac{1\cdot y_1+1\cdot y_2}{1+1}\right)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right).$$